

Spring Semester Examination 2018
Paro College of Education
Royal University of Bhutan
Paro

Module : MAT 205 (Linear Algebra)

Programme: B.Ed(S)

Level : II

Writing Time: Three Hours

Full Marks: 100

Instructions : Do not write during the first 15 minutes. Use this time for reading the questions. You will get full three hours for answering the questions. Write the answers to all the questions in the answer sheets provided by the college. Read the directions to each section and to each question carefully before answering the questions. You are allowed to carry a scientific calculator *fx-82 or fx-100* beside other writing materials. You will be provided with graph sheets.

Instructions : This paper contains FIVE questions. You can answer any FOUR questions. All questions carry 25 marks each. Marks for each question or sub question are given in the brackets.

Question 1

- a. In any $\triangle ABC$, prove using vector method that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad (6)$$

- b. Draw graph of the solution set of the following system of linear inequations: (6)

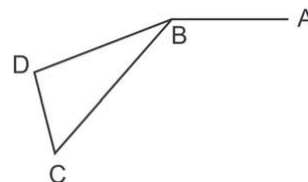
$$y \geq -x + 4, 2y \geq x - 1, 3y \leq -x + 2, x \geq 1, y \leq 6$$

- c. Using properties of determinants, solve $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ (6)

- d. The digraph represents a network of flights. (7)

- i. Create the adjacency matrix for the network.
ii. Using the adjacency matrix, find the number of one-stopover and two-stopovers from A to C.

- iii. Write all one-stopover and two-stopover pathways between A and C.



Question 2

- a. Using vector method, prove that the line segments joining the mid-points of the adjacent sides of a quadrilateral taken in order form a parallelogram. (6)

b. Solve $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$ (6)

- c. Find the Eigen values and corresponding Eigen vectors for any one of the Eigen values for the matrix (6)

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

- d. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportation cost per quintal (in Nu.)		
From/ To	A	B
D	6	4
E	3	2
F	2.5	3

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost? Solve this problem using the corner point method. (7)

Question 3

- a. Prove by vector method that in a right angled-triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. (7)
- b. Solve the following linear programming problem graphically using the Iso-cost method. (6)
- Maximize $Z = 3x + 5y$ subject to the constraints

$$-2x + y \leq 4, x + y \geq 3, x - 2y \leq 2, x \geq 0, y \geq 0$$

- c. A book store supplies three types of books priced at Nu. 350, Nu. 650 and Nu. 890, and three types of pens priced at Nu. 69, Nu. 120 and Nu. 250. The store owner wishes to mark up the price of the books by 12% and mark down the price of the pens by 10%. (6)
- Show the prices of books and pens as a suitable matrix.
 - Show the matrix obtained by marking up the price of the books by 12% and marking down the price of the pens by 10%.
 - Use matrix multiplication to calculate the new prices (correct to the nearest Ngultrums).

- d. Solve the following systems of linear equations by Cramer's rule (6)

$$x - y + z = 3, 2x + y - z = 2, -x - 2y + 2z = 1$$

Question 4

- a. If D, E, F are the mid-points of the sides of a triangle ABC, prove by the vector method that area of $\triangle DEF = \frac{1}{4}$ (area of $\triangle ABC$). (6)

- b. A retired person wants to invest an amount of up to Nu. 20,000. His broker recommends investing in two types of bonds A and B, yielding 10% and 15% return on the amount invested respectively. After some consideration, he decides to invest at least Nu.5,000 in bond A and not more than Nu. 8,000 in bond B. He also wants to invest at least as much in bond A as in bond B. What should his broker suggest if he wants to maximize his return on investments? Formulate LPP and find the maximum return using the Iso-profit method. (7)

- c. By using determinant, solve the following system of equations: (6)

$$2x - 3y - 4z = 29, -2x + 5y - z = -15, 3x - y + 5z = -11$$

- d. Using the row reduction method, obtain inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ (6)

Question 5

- a. Using vector, prove that $\cos(A+B) = \cos A \cos B - \sin A \sin B$. (7)

- b. Determine consistence of the system of the equation using determinant and find solution if exist. (6)

$$x + y + z = 9, 2x - y + z = 2, 3x + 6y + 5z = 20$$

- c. Solve the following linear programming problem graphically, using Corner Point method: (6)

Maximize $Z = 5x + 7y$, subjected to the constraints

$$x + y \leq 4, 3x + 8y \leq 24, 10x + 7y \leq 35, x \geq 0, y \geq 0$$

- d. Solve the following system of equations, using the matrix method: (6)

$$x + 2y + z = 7, x + 3z = 11, 2x - 3y = 1$$